Cognitive Based Developmental Models Used as a Link Between Formative and Summative Assessment

Karen Harris, Malcolm Bauer, and Margaret Redman
Educational Testing Service,
Rosedale Road. Princeton, NJ 08541 USA

Abstract

This paper describes the role developmental models can play in linking formative and summative assessment. We explore this issue in the context of a larger assessment design and pilot effort that is focused on middle school mathematics and English language arts, called Cognitively Based Assessment *of, for* and *as* Learning (CBAL) (Bennett & Gitomer, in press). The overall structure consists of a summative assessment system, and a formative assessment system accompanied by professional support. Research-based competency models provide a foundation for the design overall. In this context, we describe a developmental model of proportional reasoning, based upon Baxter and Junker's model in Weaver and Junker (2004) and how it was integrated into the overall design of the mathematics assessment. The developmental model helped improve the validity argument of the summative assessment by enabling us to define principled levels of competency. Because the developmental model contains research-based descriptors, for the formative assessments it provided teachers with reference points for what students can do and recommendations for areas on which they can focus to develop proficiency in a systematic way. In this way, the developmental model provided a strong link between the formative and summative assessment systems.

Introduction. Developmental models provide descriptions of skills at different levels of development. They often provide an understanding of how skills develop over time. This paper explores the use of developmental models for assessment design. We focus on the value they can play in supporting internal coherence (Gitomer & Duschl, 2007) in the alignment of formative and summative assessments. We explore this idea in the context of an assessment design effort (Bennett & Gitomer, in press) that began with the premise that cognitive models should drive the entire design and development process. The intent is to start with a research foundation about the full breadth and depth of skills in a domain, and base both summative and formative assessments upon it. In this way, the system of accountability, and the evidence teachers use for instructional decision-making in the classroom are aligned to a common and rich understanding of what it means to be competent in areas such as reading, writing, and mathematics. This paper focuses on a specific aspect of the on-going research: the pivotal role that developmental models have played in facilitating this alignment in mathematics. We describe the overall effort and discuss the design methodologies we are employing. Within this context, we describe the role that developmental models are playing in aligning the formative and summative designs in mathematics. We then provide some examples from early pilot work in middle school mathematics to illustrate the value of developmental models on the design of summative and formative assessment.

Developmental Models. Pellegrino and Goldman (2008) analyzed four contemporary mathematics curricula for K-8. One finding was that for each curriculum there was typically a sequential knowledge progression in what is assessed both within and across content and process strands, although the explicit nature of this progression is often unclear. A sequence of mathematical content has prerequisites, such as ratio precedes proportion. What is missing from the sequence is how students develop cognitively from the concept of ratio to that of proportion. A developmental model for a competency such as proportional reasoning provides this missing information. For example, a ratio is the relationship between two objects or quantities. Proportional reasoning requires considering the relationship between two relationships (the ratios). Students have to progress from the concept of an absolute change, e.g. the difference between a mother's and daughter's age over time, to a multiplicative or relative change, e.g. ratio of cups of water to cups of pancake mix as one scales a recipe for more people.

Developmental models contain qualitative descriptors of progress towards developing understanding of a domain and take into account how students typically learn. The descriptors are identifiable levels of sophistication for which evidence can be gathered from student work. The first level usually represents informal reasoning in the domain, while the highest level represents the formal conception. Even though students will not necessarily move in a sequential fashion through the model, these levels usually can be identified and they characterize enough about development to anchor key aspects of an assessment design. Levels are often described in terms of the different conceptual understandings students may have. Overall these models describe the conceptual changes students are likely to make as they move towards proficiency. A developmental model can provide teachers with a reference for what students can do and where students have to go to next to develop proficiency in a domain. Developmental models that have these properties have been proposed in different areas of mathematics. Some example models are described in Jones et al (2000) in statistical thinking, Jones et al (1997) for reasoning about probability, Weaver and Junker, (2004) in proportional reasoning, and Kalchman, Moss, & Case (2001) for rational numbers and functions.

Developmental models can offer coherence to content standards. A developmental model offers a simple structured way to monitor student progress using evidence gathered from the formative and summative assessments. A developmental model offers teachers and students clear expectations about the possible range of different levels of responses and their associated meanings, so they can work together to improve learning. Weaver and Junker (2004) describe a method for doing this type of mapping for proportional reasoning, and describe a study that provides some empirical validation of the approach. In the following sections, we describe the roles for developmental models we have found within the CBAL project. We first describe the CBAL project and the design methodology we are employing. In that context, we discuss how developmental models play a pivotal role of linking formative and summative assessment.

Cognitively-based assessment of, for, and as learning. The overall CBAL design consists of three distinct parts: a summative assessment system, a formative assessment system, and professional support to enable teachers and other stakeholders to make effective use of the systems of assessment. The summative assessment system is designed as periodic accountability assessment (PAA). Students take four task sets throughout the year. The results are then aggregated to provide an overall estimate of student competency for purposes of accountability. The key point for this paper is that intermediate results from each task set are also intended to be used to inform classroom teaching and learning. In this project, formative assessment is an ongoing process in which teachers and students use formal and informal evidence to make inferences about student competency and, based on those inferences, take actions intended to achieve learning goals (Egan, unpublished). Teachers use tasks and materials from the formative assessment system as part of their regular classroom activities to elicit evidence of student understanding to support instruction.

Using developmental models in CBAL. For the CBAL model to be effective, it is critical that the summative and formative systems are strongly linked so that both systems mutually support the teaching and learning that will produce the evidence that must be observed in student performance for that performance to be considered competent in mathematics. Linking at both the competency and evidential level is important. Pellegrino, Chudowsky, and Glaser, (2001) describe the best educational assessments as a triangle linking cognition, observation, and interpretation. By drawing from a common base of cognition (the competency model), and observation (evidence) we link two of the 3 vertices for both summative and formative assessments. In the formative system, we add the link to the interpretation part of the triangle by ensuring that the evidence is interpretable by teachers so they can adjust instruction. Using developmental models allows us to define the evidence in terms that are interpretable for instruction within the formative system from both formative and summative assessments. We employ a variant of evidence centered design or ECD (Mislevy, Steinberg, & Almond, 2003) to help represent this tight linking between cognition, observation and interpretation in the design. ECD is a methodology that has been used to bring cognitive science and psychometrics together in assessment design, and has been used previously to link formative and summative assessment (Bauer & Williamson, 2002). It provides a clear and modular way to design an assessment as a set of models that link the targets of measurement (i.e. a competency model) to the features of student performances (i.e. an evidence model) used to estimate competency, to task models that include the task characteristics needed to produce the appropriate student performances. Creating a design that consists of these linked models fills out the assessment triangle described earlier:

a) The mathematics competency model identifies the knowledge, skills, and processes we wish to measure. This model fills out the cognition component of the triangle. To construct the competency model in the CBAL project, we performed a literature review of math cognition and constructed a model that represents the knowledge, skills, and processes we believe are needed for a student's performance to be competent in middle school mathematics. We have supplemented a typical competency model with developmental models for key competencies.
b) The evidence model identifies what observable features of students' performances and responses will count as evidence of those competencies. This fills out the observation piece of the triangle. Based upon the literature review, we developed an evidence model of the different features of mathematical representations and student performances that are indicative of student competencies.

c) Task models identify the characteristics that tasks will need to have to elicit the evidence. This also fills out the observation component of the triangle in that it defines the conditions in which the observations will need to be made. A special case of a task model, called a classroom practice model, includes extra requirements such as characteristics of needed scaffolding, requirements for coverage of the content, suggested pedagogical content knowledge, and examples of student responses with a possible interpretation and suggestions and materials for "next steps" in refining instruction. Because the classroom practice model is informed by and links back to the evidence model, it is a key part of the design that allows us to delineate how the developmental models can be used to adjust instruction.

In our work, in addition to the set of three models above, we have added a model called a *discipline model*. In this model, we represent aspects of the discipline of mathematics that will be important for teaching and learning mathematics. Discipline models can include topics of study and relations among topics, curricular constraints, collaborative and social aspects of the practice of mathematics that will be important for classroom use. We have found it useful to separate aspects of the discipline itself (the discipline model) from the competencies we expect students to learn and that our assessment will measure. The CBAL mathematics competency model separates algebra, statistics & probability, geometry & measurement, and numbers & operations while part of the discipline model connects linear functions, proportional reasoning, and statistics.

Linking the three models in the assessment triangle to the discipline model creates a system of four models that drives the design of both formative and summative mathematics assessment. We partition the system of models that represents a full year of mathematics into four parts. The design process of creating the partition is documented in another manuscript by the same authors.

In describing the use of developmental models in CBAL, we will focus on the partition of the system of four models that addresses the early developmental stages of proportional reasoning. This partition includes aspects of the discipline associated with proportional reasoning, and also covers concepts associated with ratios and percentages that overlap with other content areas. The early stages of proportional reasoning are usually set in the context of comparing two parts of a whole. Figure 1 presents a graphic depiction.

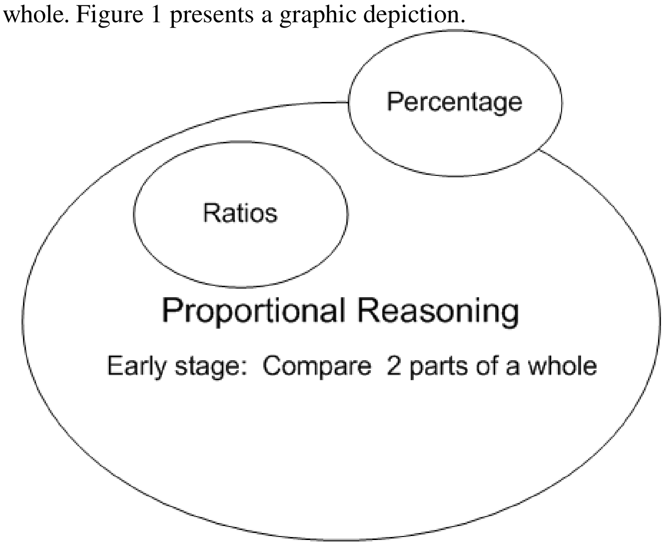


Figure 1: The portion of the discipline model

Figure 2 provides the section of the competency model addressed in this partition. It draws from several parts of the competency model including one of several content strands, specifically the content strand focused on the knowledge and skills needed for understanding and using numbers and operations. Three sub-competencies, including the competency addressing proportional reasoning are included. This portion of the competency model also includes two cross-cutting process competencies: modelling and representation, and mathematical argument and justification.

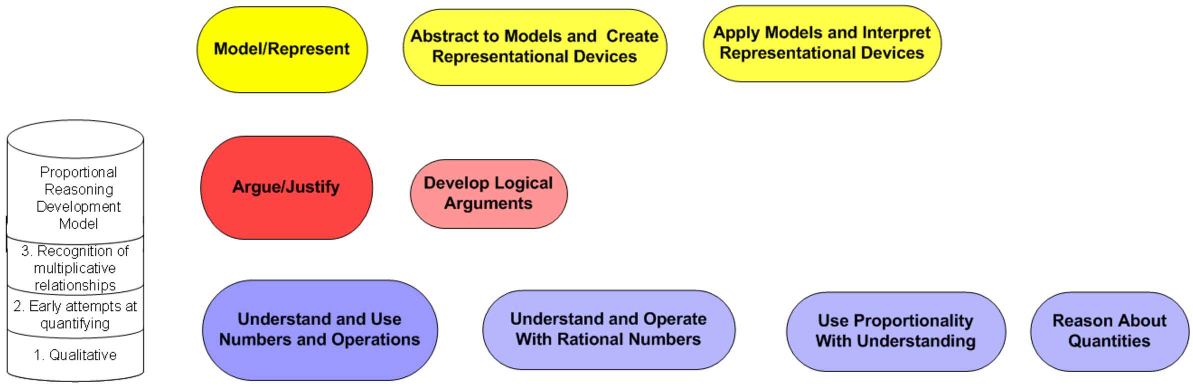


Figure 2: The portion of the mathematics competency model

The aspects of the developmental model that will be exercised in this partition are also shown in Figure 2. In this design, to characterize the development of proportional reasoning, we are using the five stage developmental model by Baxter and Junker described in Weaver and Junker (2004) and summarized in Figure 3. The Baxter and Junker model characterizes the development of proportional reasoning as a progression of stages starting with a qualitative understanding of quantity (e.g. more and less) and converging on a full understanding of proportionality as an invariant relationship between pairs of changing quantities. The five stages express the development of this deep understanding not as a continuum, but as a series of distinct models (e.g. first qualitative, then constant additive differences, followed by a multiplicative model, etc.), each explaining a wider range of situations with a more sophisticated understanding of proportionality. In this portion of the design, as expressed in Figure 2, only the bottom three of five developmental stages are included, indicating that only these three stages within the proportional reasoning developmental model that will be addressed.

Developmental Model of Proportional Reasoning

"**Qualitative**. Young students generally possess a good deal of knowledge about quantity that permits them to answer questions about more and less (e.g., which drink is sweeter?) or fairness (e.g., divide pizza or cookies so everyone gets a fair share).

Early Attempts at Quantifying. Early attempts at quantifying often involve constant additive differences (i.e., a - b = c - d) rather than multiplicative relationships. Still rely on counting up or down.

Recognition of Multiplicative Relationship. Students have the intuition that the differences change with the size of the numbers and that the change may be multiplicative in nature, but they do not necessarily realize that they need to consider the constantly increasing/decreasing difference between the related terms of each pair, that is, of each ratio. They rely on pattern matching or build-up strategies that are sometimes additive in nature. Problems with unit factors or sharing are easily solved but others may lead to additive solution strategies.

Accommodating Covariance and Invariance. Students begin to develop a change model that recognizes that while some quantities may be changing, relationships among the quantities remain invariant. However, they rely on coordinated build up, unit factor and other scalar approaches. Often strategies are context specific and not generalizable. May revert to additive strategies in unfamiliar or challenging situations.

Functional and Scalar Relationships. Students recognize the invariant nature of the relationships between pairs of changing quantities. Have generalized model for solving proportionality problems. This is not to say that they use the same strategy in every context, rather they have a repertoire of strategies and use the most efficient for a given situation."

Figure 3: Baxter and Junker's Developmental Model for Proportional Reasoning in Weaver and Junker (2004)

The evidence model specifies what evidence is being used to make inferences about the competencies being assessed. A subset of the portion of evidence used in the example proportional reasoning partition is displayed in Figure 4. The left column lists one cross-cutting process competency as well as one content competency from the full set in the competency model. The right column lists a subset of the evidence for each of these two competencies for illustration. For example, the first row states that the quality of the constraints determined by the student under which a model operates is one kind of evidence used in inferring how well students can abstract from a real world situation to a model of that situation. The constraints might be represented in a variety of ways, e.g. as a written list produced by a student. In this case, quality is a measure of the necessity of each constraint and the sufficiency of the set of constraints. (Note: Figure 4 only provides a few examples from the part of the evidence model used in the design). The subset of evidence for proportional reasoning as shown in Figure 4 will be illustrated later in the discussion of tasks used to collect these types of evidence. The developmental model influenced the choice of the types of evidence elicited in the assessments, and especially the specific scoring rules we defined to extract these types of evidence from student responses on specific tasks. Because the developmental model defines important transitions in the development of competency, it guides the selection of evidence during design. In this portion of the formative and summative assessments, we chose evidence that would help teachers identify students who were still struggling to make the transition from an additive difference model of proportionality (as in the second level of the developmental model) from those who had more fully grasped the multiplicative nature (as in level 3). Evidence in our design that supports this inference is included in Figure 4.

Competency	Evidence
Abstract to models and create representational devices	Quality of constraints under which model operates
Use proportionality with understanding	Distinction between ratio and quantity
	Description of change in mixture after changing ratio of mixture
	Comparison/contrast of 2 mixtures quantitatively

Figure 4: A subset of the evidence from the evidence model.

The next parts of the design influenced by the developmental model are the task models and classroom practice models. Figure 5 provides an extract of the specification from one of the task models. In this extract are some of the characteristics of a task that will elicit the desired evidence for the competencies and developmental model. For example, including both qualitative and basic quantitative questions enables the collection of evidence that will help distinguish between students who are at the first and second levels of the developmental model Including qualitative and basic quantitative questions as fixed elements ensures that both of these elements will be included in tasks, both formative and summative that are based on this task model.

<table>
<tr><th>Fixed Elements</th><th>Variable Elements</th></tr>
<tr><td>• The student explores the concept of proportion by comparing a mixture of 2 elements in varying proportions
• Student justifies at least one decision
• Student explores changing proportions in qualitative questions
• Student explores changing proportions in many quantitative questions
o Student reasons about ratios and proportions from data presented in tables.
o The student calculates the missing value in a ratio, given a similar ratio.
o Student extends a table.
o Student evaluates a fictional student's assertions about proportionality
o Student calculates the amount of an element needed to create a particular proportion
• Simulation tool</td><td>• Numbers of subtasks
o Number qualitative
o Number quantitative
• Developmental level represented by a fictional student's assertion about proportionality
• Students represent ratios
o as fractions
o with a colon
• Use of the simulation tool
o Aids in understanding the content
o Engages students in the task</td></tr>
</table>

Figure 5: An extract from one task model

A well-designed formative assessment will identify the gap between desired performance and a student's observed performance. The essential condition for an assessment to be used formatively is that it provides evidence that can be interpreted in a way that suggests what needs to be done next to close the gap (Shavelson, Black, Wiliam, & Coffey, 2007). A classroom practice model provides guidance to implement this essential condition. The developmental model supports the design of the classroom practice model by providing a roadmap of "next steps". When evidence is mapped onto a developmental model, what has to be done next to close the gap is within the map itself. For example, when a student provides evidence that enables teachers to hypothesize that the student has a basic understanding of the multiplicative nature of proportionality (level 3 in the developmental model), a next step to help students towards level 4 (*Accommodating Covariance and Invariance*) is that the student can start to develop a change model that recognizes that while some quantities may be changing, relationships among the quantities remain invariant. A teacher then can design instruction that combines concepts to help the student develop such a change model. Figure 6 displays an extract from the classroom practice model for a formative assessment task based on the task model in Figure 5.

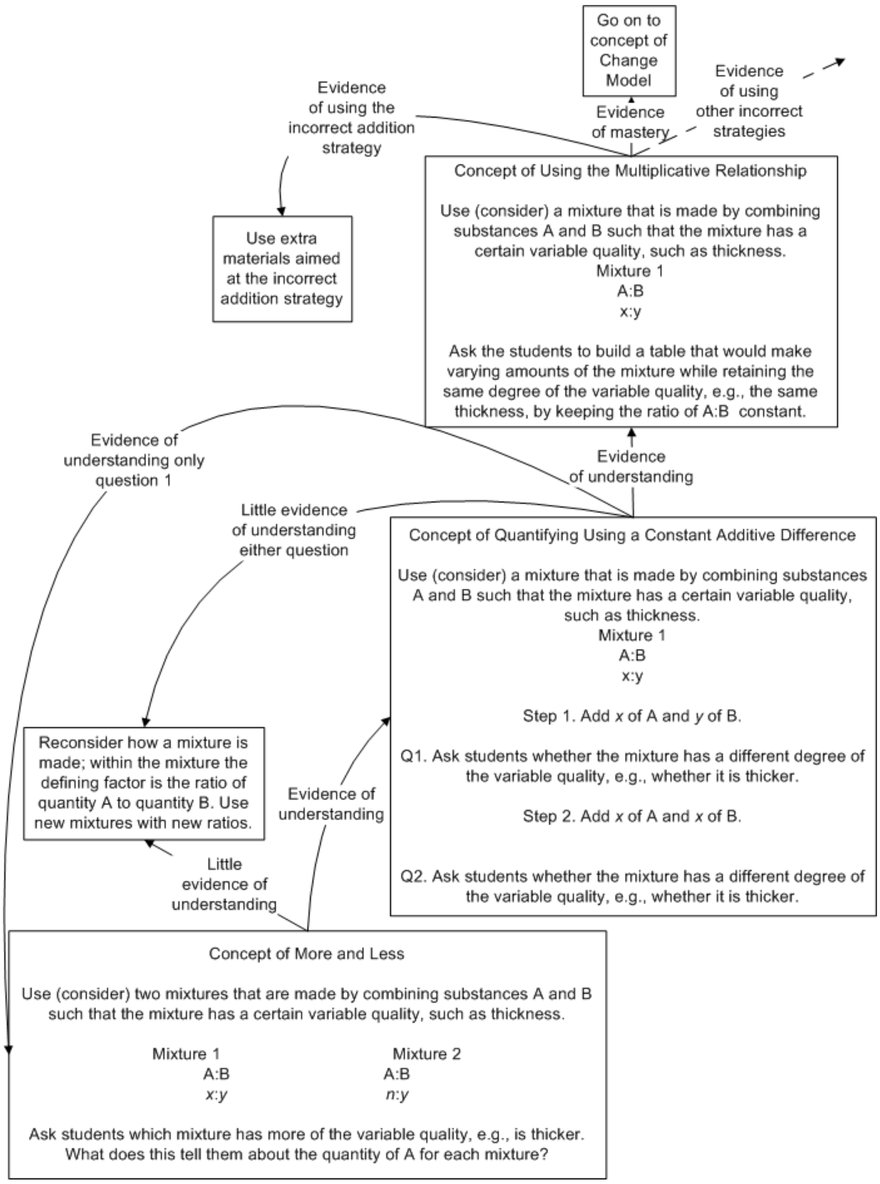


Figure 6: An extract from the classroom practice model

To illustrate the effectiveness of using the developmental model to create assessments, we provide three examples of CBAL items and student responses mapped to the proportional reasoning development model. These items were authored by Liz Marquez of Educational Testing Service for the 2007 CBAL pilot.

Two Examples from Formative Assessment. The extract from Pancake Predicament provides evidence about student knowledge in the *Qualitative l*evel and *Early Attempts at Quantifying* level in the proportional reasoning developmental model. Teachers can use the information gathered from this part of the lesson to decide on next steps in instruction. Opportunities for collecting evidence for the *Qualitative* level are presented when the students are asked to consider "thinness of the batter" in the guided questions 1 and 2 (below). Question 3 provides the opportunity to make the link between the *Qualitative* understanding and *Early Attempts at Quantifying.*

Pancake Predicament

Instructional Plan:

Present this scenario to students: I have pancake mix (show it in a clear plastic container) that contains everything but water. I lost the recipe so don't know how much water and how much mix to combine to get the right batter. So, I thought I'd try a few recipes and see what happens. But, I'll need your help to determine the right recipe.

Recipe A: Combine 1 cup of water and 2 cups of mix
Recipe B: Combine 2 cups of water and 1 cup of mix
Recipe C: Combine 3 cups of water and 4 cups of mix

Mix in order—have cards labeling amount of each ingredient. Build suspense.
Before mixing next one, elicit kids' responses to fill in a table giving ratios—part to whole and part to part. You may want to review the definition of ratio:

Recipe	Cups of Water	Cups of Mix	Ratio of cups of Water to Cups of Mix
A			
B			
C			

Now fry some pancakes from each recipe and compare them. Agree that C seems to be just right.

Discuss with students what they have just observed. Use following questions to **guide** the discussion and **assess** student understanding:

- Is the batter with the greatest amount of water the thinnest? Explain.
  Sample response: No, recipe C has more water than recipe B but B makes the thinnest batter.
- What makes a batter thin?
  Sample response: Thinness depends on the amount of mix the water is combined with. It depends on the ratio of water to mix.

- The ratio of water to mix in Recipe B is 2/1 and in C is 3/4. How can you tell which will make the thinner batter by just knowing the ratios and never seeing the batter?

  Sample response: Yes, since 2>1 and 3<4, there is more water than mix in A but more mix than water in B.
  Follow up question to Sample Response: Would the same method work if you wanted to compare recipe A to recipe C?

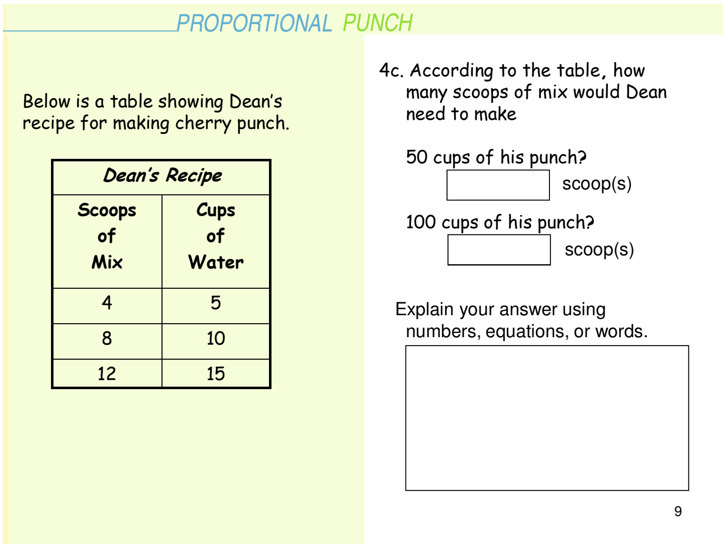
  Sample response to follow-up question: No, because in both ½ and ¾ the numerator is less than the denominator, that is 1<2 and 3<*4.*

Figure 7: Extract from the Pancake Predicament task

With respect to the evidence from Figure 3, the first question has the potential to elicit the evidence, *"Distinction between ratio and quantity"* while the second question can elicit *"Description of change in mixture after changing ratio of mixture*" and the third question can elicit "*Comparison/contrast of 2 mixtures quantitatively."*

Proportional Punch

This is a second task which was used formatively. An item from this task is shown in Figure 8 (below) with a sample, illustrative response on the right. The student's response provides evidence of recognition that while the number of cups of water and scoops of mix increases, the ratio between the two quantities remains the same. It is an example of recognizing invariance that has relied on a coordinated build-up strategy. Using the developmental model as a guide, a teacher's possible next steps would be to lead the student into setting up a ratio and finding the missing value.

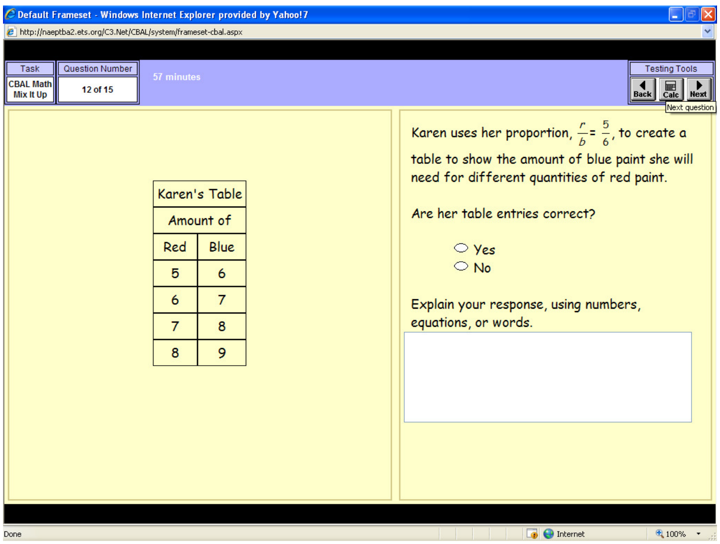


40 scoops
80 scoops

Explanation
4x10 = 40 mix
5x10 = 50 cups
40x2 = 80 mix
50 x 2 =100 cups

Figure 8: One item from the Proportional Punch task and a sample response

One item from a summative assessment. In this question, depending on the solution strategy used, the response could provide evidence of a student performing in either *Recognition of Multiplicative Relationships* or *Accommodating Covariance and Invariance* level in the case of an outstanding response.



Example Student Responses.

1. "No. because she isn't keeping the ratio of 5 to 6; she's just adding one more of each color."

3. "Yes. for one part, it would be 2.5 to 3 and each one is up to those standards."

3. "Yes; when you add 1 to each container, it keeps it."

Figure 9: CBAL item and example responses

This question was designed to identify students who recognized a multiplicative structure versus an additive structure. The values in the table were calculated as if the student had used an incorrect addition strategy. The table displays a coordinated buildup, but it is not multiplicative in nature. The students have to recognize that they need to consider whether the relationship among the pairs of values is invariant. A "yes" response is consistent with a student following an additive model rather than a multiplicative model. The explanation can provide further evidence.

Response 1. This response provides evidence that while the amount of red and blue paint in the container may be changing, relationships among the red and blue paint needs to remain invariant. This response suggests the student is operating at the *Accommodating Covariance and Invariance* level in the developmental model.

Response 2. This response provides evidence the student understands the he must find the ratio of the numbers in the first row. A "yes" response implies that the student does not realize the need to consider the constantly increasing difference between the pair of numbers in each successive row as a ratio. This suggests he had an intuition of a multiplicative relationship but did not carry this concept further.

Response 3 provides evidence of incorrect additive strategy and not a multiplicative model because the student responded "yes". This provides evidence for none of the levels of the proportional reasoning developmental model because the student is basing his response on the incorrect addition strategy. This response gives an indication to the teacher that instruction for this student needs to concentrate on the difference between using the incorrect addition strategy and an additive difference model for a multiplicative relationship. This information is from a summative assessment, but a teacher can use it formatively.

For the summative assessment, a student response can be interpreted in two ways. Firstly, a student response is matched to a score level to be aggregated with scores on other items for summative purposes. For example, both a student who explains the importance of the fact that the table in Figure 9 doesn't include a row such as 10|12 or 15|18 and a student who explains why the table entries are incorrect based on a ratio argument can earn a top level score. Secondly, the student response can be matched to a level in the proportional reasoning developmental model to be used formatively. A student who reaches the correct conclusion about the rows in Figure 9 using constant additive differences would be matched to a lower level on the proportional reasoning developmental model than one who reached that conclusion on the basis of ratio.

Conclusion. In CBAL mathematics the developmental model of proportional reasoning was integrated into the overall design of the summative and formative mathematics assessments that address the early stages of proportional reasoning. In this design process evidence that would provide information on levels of the developmental model enables the alignment of formative and summative assessment, and can help align measures for use in accountability with approaches to teaching and learning in the classroom through a common set of research based models of competency and development.. When engineering the task characteristics for inclusion in the task model, those characteristics that would elicit evidence on students' levels of the developmental model were required. When giving teachers advice on examining student work in their classroom, the developmental model drove the design and sequence of the questions. When designing the scoring guide, the proportional reasoning developmental model influenced score levels. Since CBAL's summative assessment is periodic, teachers have the opportunity to use the summative results in a formative way. Because the formative and summative assessments were designed in tandem from the same competency model and developmental model, the link between them was strong. This means that a teacher's pathway for leading students' learning towards deep understanding of mathematics is clear and can be approached in a systematic way.

References

Bauer, M. I. and Williamson, D. E. (2002). Using evidence centered design to align formative and summative assessment. *Proceedings of the workshop on evidence centered design*, (pp 87-96). Conference on intelligent tutoring systems, San Sebastian, Spain.

Bennett, R. E., & Gitomer, D. H. (in press). Transforming K-12 assessment. In C. Wyatt-Smith & J. Cumming (Eds.), *Assessment issues of the 21st Century.* New York, NY: Springer Publishing Company.

Kalchman, M., Moss, J., and Case, R. (2001). Psychological models for the development of mathematical understanding: Rational numbers and functions. In S. M. Carver and D. Klahr (Eds.) *Cognition and instruction: 25 years of progress*. Lawrence Erlbaum Associates: New Jersey.

Egan, T. (Unpublished manuscript not yet submitted for publication). *Overview of CBAL Formative Assessment.*

Gitomer, D. H., & Duschl, R. A. (2007). Establishing multilevel coherence in assessment. In P. A. Moss (Ed.), *Evidence and Decision Making. The 106th yearbook of the National Society for the Study of Education, Part I.* (pp. 288-320). Chicago: National Society for the Study of Education.

Jones, G. A., Langrall, C. W., Thornton, C. A., & Mogill, A. T. (1997). A framework for assessing and nurturing young children's thinking in probability. *Educational Studies in Mathematics*, 32, 101-125.

Jones, G. A., Langrall, C. W., Thornton, C. A, Mooney, E. S. (2000). A framework for characterizing children’s statistical thinking. *Mathematical thinking and learning,* 2(4), 269-307.

Mislevy, R. J., and Risconcente, M.M. (2006) Evidence centered assessment design. In S. M. Downing and T. M. Haladyna (Eds.) *Handbook of test development*. London: Routledge.

Mislevy, R. J., Steinberg, L. S., & Almond, R. G. (2003). On the structure of educational assessments. *Measurement: Interdisciplinary Research and Perspectives*, 1, 3–62.

Pellegrino, J., Chudowsky, N., & Glaser, R. (2001). *Knowing what students know: the science and design of educational assessment*. Washington, D.C.: National Academy Press.

Pellegrino, J., Goldman, S. (2008). Beyond the Rhetoric: Realities and Complexities of Integrating Assessment into Classroom Teaching and Learning. In Carol Dwyer (Ed) *The Future of Assessment: Shaping Teaching and Learning.* Philadelphia, PA: Lawrence Erlbaum Associates.

Risconcente, M.M, Mislevy, R. J, Hamel, L., and the PADI Research Group (2005). *An introduction to PADI task templates*. (PADI Technical Report No. 3). Menlo Park, CA: SRI International.

Shavelson, R., Black, P., Wiliam D. & Coffey, J. (in press). On Linking Formative and Summative Functions in the Design of Large-Scale Assessment Systems. *Educational Evaluation and policy Analysis*.

Weaver, R., & Junker, B. W. (2004). *Model specification for cognitive assessment of proportional reasoning* (Department of Statistics Technical Report No. 777). Pittsburgh, PA: Carnegie Mellon University.